Mitigating the effects of multicollinearity using principal components analysis

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This is standard fare, but it is easy to get lost in the details...

Suppose that we have a matrix X of n observations on k factors.

Suppose that W is the k by k matrix of the eigenvectors of the k by k correlation matrix V of X.

Since V is symmetric and positive definite, all its eigenvalues are real and positive.

Then the n by k matrix P of the principal components of V is such that: P=XW.

So if we regress on P instead of regressing on X, we obtain a vector of k coefficients γ, say, such that

$$P\gamma = (XW)\,\gamma = X\,(W\gamma)$$

Since P is orthogonal, the regression on P is well behaved, unless the rank of V is less than k.

But since the factors are multicollinear, $\beta = W\gamma$ is not well defined.

In order to approximate X, we can use the matrix $W_{(j)}$ of the first j eigenvectors of V, which define the first j columns of P, renamed $P_{(j)}$.

Since W is orthonormal, we have that $W^{-1} = W^{'}$, so that $X = PW^{'}$. The corresponding approximation for X would be

$$X_{(j)} = P_{(j)}W_{(j)}^{'}$$

This approximation could be such that the k columns of matrix $X_{(j)}$are no longer multicollinear.

Accordingly, if we name $\gamma_{(j)}$the first j coefficients of the vector γ, we have that

$$P_{(j)}\gamma_{(j)} = \left(X_{(j)}W_{(j)}\right)\gamma_{(j)} = X_{(j)}\left(W_{(j)}\gamma_{(j)}\right)$$

Thus, if we define $\beta_{(j)} = W_{(j)}\gamma_{(j)}$, we have a vector of k well behaved coefficients, in principle.

The proportion of the variance of X explained by the first j eigenvectors of V is given by the proportion of the cumulative sum of the corresponding eigenvalues of V to the sum of all eigenvalues of V. Since a few of the largest eigenvalues of V form the largest proportion of the sum of all eigenvalues of V when multicollinearity is a problem, we must choose the j eigenvectors corresponding to the largest j eigenvalues of V to approximate X.

References

[1] Alexander, C., "Practical Financial Econometrics", Wiley, 2008, pp. 23-27